# Exercise Set #3
## "Discrete Mathematics" (2025)

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*Exercise 7 is to be submitted on Moodle before 23:59 on March 10th, 2025*

**E1.** **(a)** How many positive integers are there that divide $10^{40}$ or $20^{30}$ ?

**(b)** How many positive integers less than or equal to 385 are there such that they are not divisible by neither of the following numbers: $5, 7, 11$ ?

**E2.** Determine the number of permutations of the set $[n]$

**(a)** with exactly one fixed point, and

**(b)** with exactly $k$ fixed points.

**E3.** How many functions $f : [n] \to [n]$ are there that are nondecreasing? That is, they satisfy $i < j \Rightarrow f(i) \leq f(j)$.

**Hint:** A non decreasing function is determined by the differences $f(i) - f(i-1)$ with $f(0) = 1$.

**E4.** Assume that $k > n$. Prove that the number of surjective functions from $[k]$ to $[n]$ is given by

$$\sum_{j=0}^{n} \binom{n}{j} (-1)^j (n-j)^k$$

**E5.** Prove the following.

**(a)** If $\varphi(n)$ divides $n-1$ then $n = p_1 \cdot \ldots \cdot p_r$ where $p_i \neq p_j$ for $i \neq j$.

**(b)** $\varphi(n)$ is even for $n \geq 3$.

**(c)** For every natural number $n$, we get

$$\sum_{d|n} \varphi(d) = n$$

where the sum is taken over all divisors $d$ that divide $n$.

**E6.** **(a)** Suppose we have $\mu$ identical particles and $n$ distinct energy levels, with $n \geq \mu$. In how many ways can we distribute the particles among the levels so that there is at most one particle per level?

**(b)** Suppose we have $\mu$ distinct particles and $n$ distinct energy levels, with $\mu \geq n$. In how many ways can we distribute the particles among the levels so that there is at least one particle per level?

**E7.** (**Exercise to submit**)
Thomas fires 82 shots at a square target with a side length of 90 cm. He is quite skilled at shooting, as all of his shots hit the target. He claims that at least two bullet holes are less than 15 cm apart. Is Thomas correct?